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Generalized Chaplygin Gas Model as a New Agegraphic Dark Energy in Non-flat Universe

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Abstract In this paper we consider a correspondence between the new agegraphic dark energy density and generalized Chaplygin gas energy density in non-flat FRW universe. Then we reconstruct the potential and the dynamics of the scalar field which describe the generalized Chaplygin cosmology.

Keywords Generalized Chaplygin gas · Agegraphic dark energy · Non-flat universe · Equation of state

1 Introduction

Many cosmological observations, such as SNe Ia [1–3], WMAP [4, 5], SDSS [6–8], Chandra X-ray observatory [9], etc., reveal that our universe is undergoing an accelerating expansion. To explain this cosmic positive acceleration, mysterious dark energy has been proposed. There are several dark energy models which can be distinguished by, for instance, their equation of state (EoS) ($w = \frac{p_{de}}{\rho_{de}}$) during the evolution of the universe. Astrophysical data also indicate that w lies in a very narrow strip close to -1. The case $w = -1$ corresponds to the cosmological constant. For w less than -1 the phantom dark energy [10–14] is observed, and for w more than -1 (but less than $\frac{-1}{3}$) the dark energy is described by quintessence [15–18]. More ever, the analysis of the properties of dark energy from recent observational data mildly favor models of dark energy with ω crossing -1 line in the near past. So, the phantom phase equation of state with $\omega < -1$ is still mildly allowed by observations. Most of dark energy models treat scalar field(s) as dark component(s) with a dynamical equation of state. So far, a large class of scalar-field dark energy models have been studied, including quintessence [15–18], K-essence [19], tachyon [20, 21], phantom [10–14], ghost condensate [22, 23] and quintom [24–31], and so forth. But we should note that the mainstream viewpoint regards the scalar field dark energy models as an effective description of an underlying theory of dark energy. In addition, other proposals on dark energy include interacting dark energy models [32–35], braneworld models [36, 37], and Chaplygin

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gas models [38], etc. One should realize, nevertheless, that almost these models are settled at the phenomenological level, lacking theoretical root.

In a very interesting paper Kamenshchik, Moschella, and Pasquier [38] have studied a homogeneous model based on a single fluid obeying the Chaplygin gas equation of state

$$P = \frac{-A}{\rho}, \tag{1}$$

where P and ρ are respectively pressure and energy density in comoving reference frame, with $\rho > 0$; A is a positive constant. This equation of state has raised a certain interest [39–42] because of its many interesting and, in some sense, intriguingly unique features. Some possible motivations for this model from the field theory points of view are investigated in [43, 44]. The Chaplygin gas emerges as an effective fluid associated with d-branes [45, 46] and can also be obtained from the Born-Infeld action [47]. There are some generalization of the EoS (1). In the generalized Chaplygin gas approach [47], the equation of state to (1) is generalized to

$$P_\Lambda = \frac{-A}{\rho_\Lambda^\alpha}. \tag{2}$$

The above equation of state leads to a density evolution as

$$\rho_\Lambda = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}. \tag{3}$$

WMAP data show that α in the above EoS is non-zero and the data is well-fitted with the generalized Chaplygin gas than any other EoS [48–50].

An approach to the problem of DE arises from the holographic principle that states that the number of degrees of freedom related directly to entropy scales with the enclosing area of the system. It was shown by 't Hooft and Susskind [51] that effective local quantum field theories greatly overcount degrees of freedom because the entropy scales extensively for an effective quantum field theory in a box of size L with UV cut-off Λ. As pointed out by [52], attempting to solve this problem, Cohen et al. showed [53] that in quantum field theory, short distance cut-off Λ is related to long distance cut-off L due to the limit set by forming a black hole. In other words the total energy of the system with size L should not exceed the mass of the same size black hole i.e. $L^3\rho_\Lambda \le LM_p^2$ where ρ_Λ is the quantum zero-point energy density caused by UV cutoff Λ and M_P denotes Planck mass ($M_p^2 = 1/8\pi G$). The largest L is required to saturate this inequality. Then its holographic energy density is given by $\rho_\Lambda = 3c^2M_p^2/L^2$ in which c is free dimensionless parameter and coefficient 3 is for convenience. On the basis of the cosmological state of the holographic principle, proposed by Fischler and Susskind [54], a holographic model of dark Energy (HDE) has been proposed and studied widely in the literature [55–66]. More recently a new dark energy model, dubbed agegraphic dark energy has been proposed [67] (see also [68, 69]), which takes into account the Heisenberg uncertainty relation of quantum mechanics together with the gravitational effect in general relativity. Following the line of quantum fluctuations of spacetime, Karolyhazy [70–72] proposed that the distance t in Minkowski spacetime cannot be known to a better accuracy than $\delta t = \lambda t_p^{2/3} t^{1/3}$, where λ is a dimensionless constant of order unity. Based on Karolyhazy relation, Maziashvili proposed that the energy density of metric fluctuations of Minkowski spacetime is given by [73, 74]

$$\rho_\Lambda \sim \frac{1}{t_p^2 t^2} \sim \frac{M_p^2}{t^2}, \tag{4}$$

where t_p is the reduced Planck time, and M_p is the Planck mass.

In the present paper, we suggest a correspondence between the new agegraphic dark energy scenario and the generalized Chaplygin gas dark energy model. We show this new agegraphic description of the generalized Chaplygin gas dark energy in non-flat FRW universe and reconstruct the potential and the dynamics of the scalar field which describe the generalized Chaplygin cosmology.

2 Agegraphic Generalized Chaplygin Gas Model

Here we consider the Friedmann-Robertson-Walker universe with line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right). \tag{5}$$

The first Friedmann equation is given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}[\rho_\Lambda + \rho_{\rm m}]. \tag{6}$$

Define as usual

$$\Omega_{\rm m} = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_{\rm m}}{3M_p^2H^2}, \qquad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}} = \frac{\rho_\Lambda}{3M_p^2H^2}, \qquad \Omega_k = \frac{k}{a^2H^2}. \tag{7}$$

We know that the scalar field models of dark energy are effective description of such a theory, we should be capable of using the scalar-field model to mimic the evolving behavior of the new agegraphic dark energy and reconstructing this scalar-field model according to the evolutionary behavior of new agegraphic dark energy. So following [78] (see also [79–82]) we assume that the origin of the dark energy is a scalar field ϕ, so

$$\rho_\phi = \frac{1}{2}\dot\phi^2 + V(\phi) = \rho = \left[A + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}, \tag{8}$$

$$P_\phi = \frac{1}{2}\dot\phi^2 - V(\phi) = P = \frac{-A}{[A + \frac{B}{a^{3(1+\alpha)}}]^{\frac{\alpha}{1+\alpha}}}. \tag{9}$$

Then, one can easily derive the scalar potential and kinetic energy term as

$$V(\phi) = \frac{A + \frac{B}{2a^{3(1+\alpha)}}}{[A + \frac{B}{a^{3(1+\alpha)}}]^{\frac{1}{1+\alpha}}}, \tag{10}$$

$$\dot\phi^2 = \frac{B}{a^{3(1+\alpha)}[A + \frac{B}{a^{3(1+\alpha)}}]^{\frac{\alpha}{1+\alpha}}}. \tag{11}$$

Now we suggest a correspondence between the new agegraphic dark energy scenario and the generalized Chaplygin gas dark energy model. The original agegraphic dark energy density has the form (4) where t is chosen to be the age of the universe

$$t = \int_0^a \frac{da}{aH}. \tag{12}$$

Thus, the energy density of the agegraphic dark energy is given by

$$\rho_\Lambda = 3n^2 M_p^2 t^{-2}, \tag{13}$$

where the numerical factor $3n^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe. The original agegraphic dark energy model has some difficulties [67]. According to the new agegraphic dark energy we have following relation for energy density [75–77]

$$\rho_\Lambda = 3n^2 M_p^2 \eta^{-2}, \tag{14}$$

where η is conformal time, and given by

$$\eta = \int \frac{dt}{a} = \int \frac{da}{a^2 H}. \tag{15}$$

Using definitions $\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}$ and $\rho_{cr} = 3M_p^2 H^2$, we get

$$H\eta = \frac{n}{\sqrt{\Omega_\Lambda}}. \tag{16}$$

The conservation equations for CDM and dark energy are given respectively as

$$\dot{\rho_m} + 3H\rho_m = 0, \tag{17}$$

$$\dot{\rho}_\Lambda + 3H(1 + w_\Lambda)\rho_\Lambda = 0. \tag{18}$$

Using (14)–(18)we find that the equation of motion for Ω_Λ is given by

$$\frac{d\Omega_\Lambda}{da} = \frac{\Omega_\Lambda}{a}(1 + \Omega_k - \Omega_\Lambda)\left(3 - \frac{2}{n}\frac{\sqrt{\Omega_\Lambda}}{a}\right), \tag{19}$$

or in another term as

$$\frac{d\Omega_\Lambda}{dz} = -\Omega_\Lambda(1 + \Omega_k - \Omega_\Lambda)\left(\frac{3}{1+z} - \frac{2}{n}\sqrt{\Omega_\Lambda}\right), \tag{20}$$

where $z = \frac{1}{a} - 1$ is the redshift of the universe. Using (14), (16), (18), one can obtain the equation of state as

$$w_\Lambda = -1 + \frac{2}{3n}(1+z)\sqrt{\Omega_\Lambda}. \tag{21}$$

If we establish the correspondence between the agegraphic dark energy and generalized Chaplygin gas energy density, then using (3), (14) we have

$$B = (3n^2 a^3 M_p^2 \eta^{-2})^{(1+\alpha)} - A^{3(1+\alpha)}. \tag{22}$$

Also using (2), (3), (21) one can write

$$w = \frac{P}{\rho} = \frac{-A}{\rho^{1+\alpha}} = \frac{-A}{(3n^2 M_p^2 \eta^{-2})^{(1+\alpha)}} = w_\Lambda = -1 + \frac{2}{3n}(1+z)\sqrt{\Omega_\Lambda}. \tag{23}$$

Substitute B in the above equation, we obtain following relation for A:

$$A = (3n^2 M_p^2 \eta^{-2})^{(1+\alpha)} \left[1 - \frac{2}{3n}(1+z)\sqrt{\Omega_\Lambda} \right]. \tag{24}$$

Then B is given by

$$B = \frac{2}{3n}(3n^2 M_p^2 \eta^{-2})^{(1+\alpha)}(1+z)\sqrt{\Omega_\Lambda}. \tag{25}$$

Now we can rewrite the scalar potential and kinetic energy term as following

$$V(\phi) = (3n^2 M_p^2 \eta^{-2})^{\alpha} \left(1 - \frac{(1+z)\sqrt{\Omega_\Lambda}}{3n} \right), \tag{26}$$

$$\dot{\phi} = \frac{M_p}{\eta}\sqrt{2n(1+z)\sqrt{\Omega_\Lambda}}. \tag{27}$$

Considering $a = \frac{1}{1+z}$, we have

$$\dot{\phi} = \frac{-H}{a}\frac{d\phi}{dz} = \frac{-H}{a}\phi' = -H(1+z)\phi'. \tag{28}$$

Then derivative of scalar field ϕ with respect to z is as

$$\phi' = -M_p \sqrt{\frac{2}{n(1+z)}\Omega_\Lambda^{3/2}}. \tag{29}$$

Consequently, we can easily obtain the evolutionary form of the field

$$\phi(z) = -M_p \sqrt{\frac{2}{n}} \int_0^z \Omega_\Lambda^{3/4}(1+z)^{-1/2} dz. \tag{30}$$

3 Conclusions

In this paper we have associated the new agegraphic dark energy in FRW universe with a scalar field which describe the generalized Chaplygin cosmology. We have shown that the new agegraphic dark energy can be described by the scalar field in a certain way. Then a correspondence between the agegraphic dark energy and generalized Chaplygin gas model of dark energy has been established, and the potential of the agegraphic scalar field and the dynamics of the field have been reconstructed.

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